

Consider a pivot style choice experiment where the choices are pivoted off of each participant's expectation of the future state. Let these states be described by a vector of attribute levels $[x_1, \dots, x_k, x_c] = \mathbf{x} \in X$, where X is the set of all possible combinations of attribute levels and the element x_c be a cost. Let the participants beliefs about the future be represented by a probability distribution $f(\mathbf{x}|\boldsymbol{\theta}, \mathbf{z}_i)$ where $\boldsymbol{\theta}$ is a vector of parameters for the distribution and $\mathbf{z}_i$ is a vector of characteristics of the participant that impact their beliefs.

The experiment proceeds by offering the participant a set of alternatives from which they choose one that they believe is the most likely, absent the specific policy being considered in the choice experiment. Let the set A be the set of alternatives offered to the participant as possible futures, with $x_c = 0$ for all elements in A . Let α be a partition of X such that $\bigcup \alpha = X$ and each $\mathbf{x} \in A$ is in only one element of α and no element of α does not contain a vector in A . We can then define the probabilities across $\mathbf{x} \in A$ as

$$\Pr(\mathbf{x}_{i0}) = \int_{\alpha(\mathbf{x}_{i0})} f(\mathbf{x}|\boldsymbol{\theta}, \mathbf{z}_i) d\mathbf{x}$$

where $\mathbf{x}_{i0}$ is the future chosen by the participant. A representation of this probability would be a multivariate ordinal distribution, if the underlying attributes are continuous and the offered attribute levels are ordinal.

With the future chosen by the participant $\mathbf{x}_{i0}$, the set of possible choices that could be offered to the participant is

$$B(\mathbf{x}_{i0}) = \{\mathbf{x} \in A; \mathbf{x} > \mathbf{x}_{i0} \rightarrow x_c \in A_c\}$$

where $>$ indicates that at least one element in $\mathbf{x}$ is greater than its corresponding element in $\mathbf{x}_{i0}$, and x_c is a cost value from the set of possible cost values A_c included in the experiment. Note that since $x_c = 0$ for all elements in A , the operation $>$ will choose those elements of A where at least one of the non-cost attributes is better than in $\mathbf{x}_{i0}$. Participants are offered a subset of the choices in $B(\mathbf{x}_{i0})$.

Assume that the utility the participant expects from any vector $\mathbf{x}$ is

$$U_i(\mathbf{x}_{ij}) = V_i(\mathbf{x}_{ij}) + \epsilon_{ij}$$

where the utility is made up of an observed component $V_i(\mathbf{x}_{ij})$ and an unobserved component ϵ_{ij} . For any single choice task, with $\mathbf{x}_{ij} \in B(\mathbf{x}_{i0})$, the

participant will choose the alternative if

$$V_i(\mathbf{x}_{ij}) + \epsilon_{ij} > V_i(\mathbf{x}_{i0}) + \epsilon_{i0}$$

with the probability of the participant choosing $\mathbf{x}_{ij}$ when their forecast future is $\mathbf{x}_{i0}$ being

$$\Pr(\mathbf{x}_{ij}|\mathbf{x}_{i0}) = \Pr[V_i(\mathbf{x}_{ij}) - V_i(\mathbf{x}_{i0}) > \epsilon_{i0} - \epsilon_{ij}]$$

The probability that participant i chooses $\mathbf{x}_{ij}$ is therefore

$$\Pr(\mathbf{x}_{ij}) = \Pr(\mathbf{x}_{i0}) \times \Pr(\mathbf{x}_{ij}|\mathbf{x}_{i0})$$

which allows for the form of the observed contribution to the participants utility or the error to be conditional on the future predicted by the participant. Note that with a pivot design where $\mathbf{x}_{i0}$ is the common status quo across the set of choices offered to the participant. The unobserved portion of the utility, ϵ_{i0} , is common across the choice set as well, and needs to be accounted for in the estimation (Train and Wilson, 2008).

Specific to the experiment reported on here, the choices are described by three environmental attributes and one cost attribute. In choosing a predicted future, the participant selects sequentially the future level of the three environmental attributes from a menu that contains the levels of the attributes that will be used in the choice experiment. Assume that the participants' beliefs $\mathbf{x}_{i0}^*$ are a draw from a quatravariate continuous distribution with the cost element degenerate as $x_c = 0$. Consistent with multivariate ordinal distributions, we assume a set of cutoff variables τ_{jk} that divide the domain of each environmental attribute into k^j regions, where k^j is the number of levels of attribute j offered to the participant, and the subscript k indexes the attribute. We further assume that the beliefs of participants i for the future level of attribute k can be represented as

$$x_{i0k}^* = \gamma_{0k} + \mathbf{z}_i' \boldsymbol{\gamma}_k + \varepsilon_{0k}$$

where $\mathbf{z}_i$ is a vector of characteristics for the participant, $\boldsymbol{\gamma}_k$ captures the influence that individual specific characteristics have on the beliefs, γ_{0k} is an intercept, and ε_{0k} is an error term. Correlation between the participants beliefs across attributes is likely, and captured through the distribution of the vector $\boldsymbol{\varepsilon}_0$. Estimation methods for the multivariate normal distribution are straightforward, although computationally intensive. Methods for the multivariate logistic

distribution have been developed and implemented by several authors (O’Brien and Dunson, 2004; Nooraee et al., 2016; Hirk et al., 2019). The output of this estimation includes for each participant a probability for each possible future $\mathbf{x} \in A$ that they were offered to choose among. This provides $\Pr(\mathbf{x}_{i0})$.

It is common to define the observed portion of the utility function as

$$V_i(\mathbf{x}) = \beta_0 + \mathbf{x}'\boldsymbol{\beta} + \epsilon$$

where individual heterogeneity can be introduced through interactions between β_0 and/or $\boldsymbol{\beta}$ with individual specific characteristics or through random parameters that themselves may have distributions that depend on levels of the individual specific characteristics. In this form, the utility depends on the level of the attributes in the chosen alternative and in the participant forecast, and possibly the individual specific characteristics. It does not depend on the size of the choice set from which the choices the participant saw were drawn from. The probability that the alternative is chosen is therefore

$$\Pr(\mathbf{x}_{ij}|\mathbf{x}_{i0}) = \Pr(\beta_0 + \mathbf{x}_{ij}'\boldsymbol{\beta} - \mathbf{x}_{i0}'\boldsymbol{\beta} > \epsilon_{i0} - \epsilon_{ij})$$

With the usual assumption that the error has an extreme value distribution, estimation as a multinomial logit follows. When $\mathbf{x}_{i0}$ serves as a pivot for multiple choices by each participant, then estimation needs to account for the presence of the common error ϵ_{i0} in all choices by the same participant (Train and Wilson, 2008).

The final likelihood is therefore the product of the probability that the participant chose $\mathbf{x}_{i0}$ as a prediction for the future and the probability that the participant chose the alternative offered, $\mathbf{x}_{ij}$, instead of their predicted future $\mathbf{x}_{i0}$. It is in effect a weighted multinomial logistic regression where the weights are the probability that the participant chose a particular future from the set on offer.

References

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